Weighing the World, the Cavendish Experiment

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This manual adapted from **Determining the gravitational constant with the gravitation torsion balance after Cavendish** [1]. *Read through this entire lab manual before beginning the experiment.*

WARNING

The torsion balance apparatus is *extremely* sensitive, and has been very carefully aligned for you. **Absolutely do not touch** the laser or any part of the apparatus, with the exception of the swivel plate of the two large spheres.

Additionally, do not look directly into the laser beam.

I. INTRODUCTION

According to Newtonian theory, the gravitational force between two objects of mass m_1 and m_2 separated by a distance r is

$$F = \frac{Gm_1m_2}{r^2} \tag{1}$$

In our case this force is very weak, on the order of 10^{-9} N. Nevertheless it can be measured using an extremely sensitive torsion balance. In turn this measurement can be used to determine the gravitational constant G. This experiment was first carried out by Henry Cavendish in the 1790s, using high precision vernier scales instead of laser light to measure deflection angles.

At the time of his original experiment, the familiar formulation of Equation 1 was not yet standard. Cavendish did not measure G as we know it, but instead sought to measure the density of the Earth. His data give a value of $\rho = 5.448$ g/cm^3. After a little algebra, this is converted into $G = 6.74 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, remarkably close to the current accepted value of $G = 6.673 \times 10^{-11}$ m^3 kg^{-1} s^{-2} [2].

In this experiment, we attempt to experimentally measure G using a scaled down version of Cavendish's famous experiment. This apparatus consists of a torsion pendulum inside of a plastic case, a laser, and a measuring screen. The torsion pendulum is a light transverse beam of length $2d$ suspended by a thin torsion string. At the center of the beam is a mirror, and at each end of the beam is a small sphere of mass m_2. These two balls are attracted to two larger spheres of mass m_1.

The movement of the small spheres is observed using a laser pointer. The laser is incident on the mirror and is reflected onto the measuring screen. By measuring linear displacement of the laser light on the screen, the deflection angle of the pendulum can be determined, and hence G can be calculated.

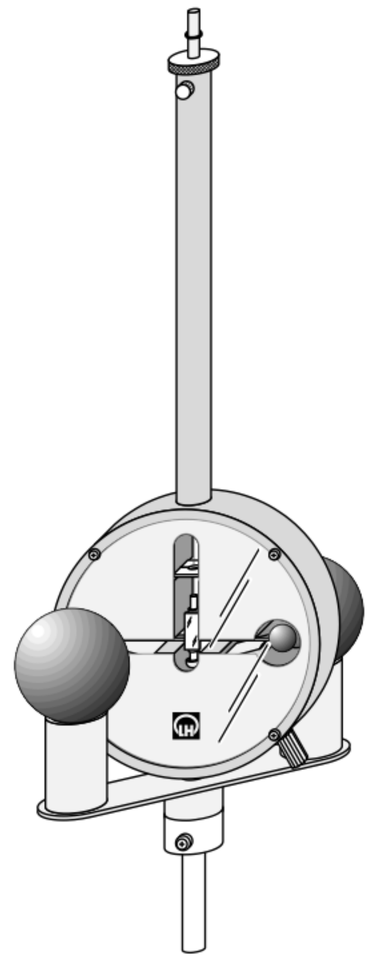

FIG. 1: Torsion pendulum apparatus

II. TORSION PENDULUM

The gravitational force is given in Equation 1. The moment of momentum acting on the torsion rod when the two large spheres are in position 1 is

$$M_1 = 2Fd = 2G\frac{m_1 m_2}{r^2}d \quad (2)$$

This moment is counteracted by the restoring moment of the torsion string. The torsion pendulum executes damped oscillations around, and eventually assumes, an equilibrium position θ_1. When the large spheres are swiveled to position 2 the moment of momentum acting on the torsion rod is reversed, with $M_2 = -M_1$. The torsion rod undergoes damped oscillations around and settles at a second equilibrium position θ_2

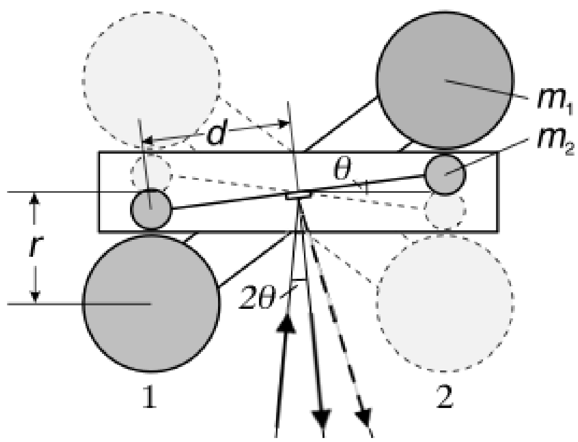

FIG. 2: The two positions of the large spheres.

For the difference between the two moments of momentum, we can say

$$D(\theta_1 - \theta_2) = M_1 - M_2 = 2M_1 \quad (3)$$

This angular quantity D can be determined from the oscillation period T and the moment of inertia I of the torsion pendulum

$$D = I\frac{4\pi^2}{T^2} \quad (4)$$

The moment of inertia I is approximated as the moment of inertia from the two balls, $I = 2m_2 d^2$.

Combining Equations 1, 2, 3, and 4 we can obtain

$$G = \frac{2dr^2\pi^2}{m_1 T^2}(\theta_1 - \theta_2) \quad (5)$$

A. Antitorque correction

The above derivation only takes into account the contribution of nearest large sphere to the small spheres on the torsion rod. *Both* large spheres have a significant contribution to the moment, so the attractive force of the opposing sphere should not be neglected.

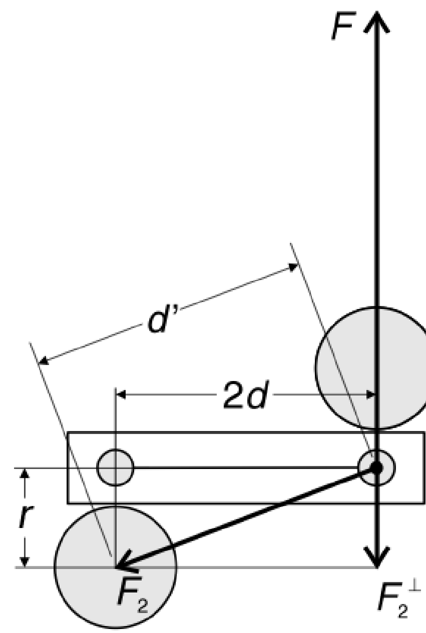

FIG. 3: Diagram showing the antitorque correction.

Based on the above calculation for the moment of momentum acting on the torsion rod, we now have

$$M_1 = 2(F + F_2^{\perp})d \quad (6)$$

where $F_2^{\perp} = -F_2 \cdot r/d'$ is the component of the force $F_2 = F \cdot r^2/d'^2$ which is acting perpendicular to the transverse beam.

This makes the gravitational constant G is greater than previously calculated by a correction factor

$$K = \frac{F}{F + F_2^{\perp}} = \frac{1}{1 - \frac{r^3}{d'^3}} \quad (7)$$

B. Measuring deflection angles

The basic geometry of the apparatus is shown in Figure 4. What we want to measure are the deflection angles θ_1 and θ_2 of the light beam from the central equilibrium position p_0.

Since we are only concerned with the deflection from this position, it is not necessary to determine the absolute deflection angles. For each position p_i, the distance along the screen from the laser is $L_i = L_0 \pm d_i$. The total deflection angle of this beam is thus given by

$$2\theta_i = \tan^{-1}\left(\frac{L_i}{L_0}\right) \quad (8)$$

III. METHODS

Upon first entering the lab, visually inspect to ensure that the laser is active and that it is aligned with the screen. DO NOT TOUCH ANYTHING. Ensure that the large spheres are in their proper place on the apparatus. If the swivel arm is already in position 1 or 2 as seen in Figure 2 and the pendulum is settled, record the position p_i of the laser on the screen.

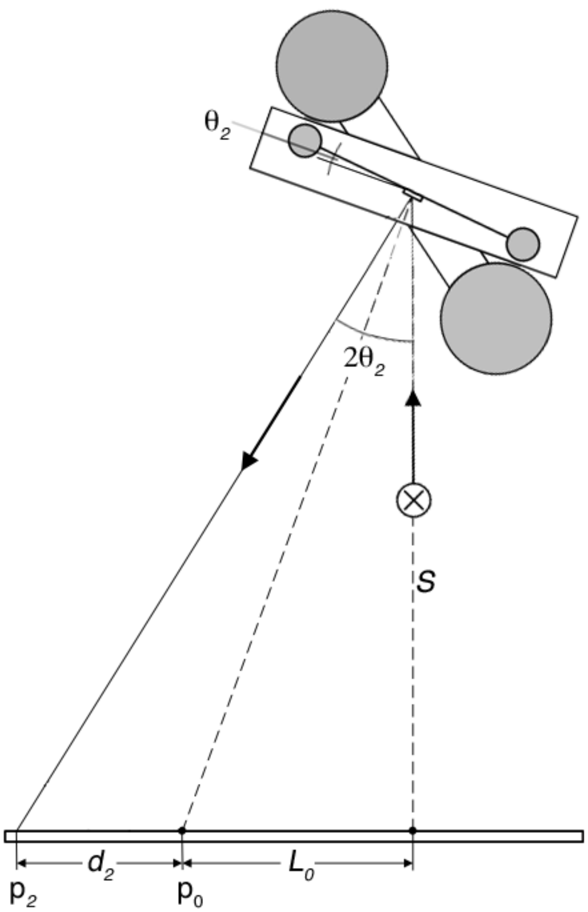

FIG. 4: Geometry of the apparatus. The distances to be measured are S, L_0, d_1 and d_2. (Diagram shows position 2. Swivel to position 1 to measure d_1.). Note that the middle dashed line is the laser at equilibrium position, NOT the mirror normal.

If the arm is not in one of those positions, carefully move it into position 1. As you swivel the large spheres, be very careful not to bump the pendulum housing. In position, the large spheres should be as close as possible to the plastic face of the apparatus without actually touching. Once moved, leave the apparatus to settle. This takes over two hours, so you may want to talk to the instructor about leaving for the day and coming back later. Once settled, record the position of the laser on the screen. (p_i in Figure 4)

Next, carefully move the swivel arm into the other position. The pendulum will execute damped oscillations around the new equilibrium point. Using a computer or digital camera, track the movement of the laser along the screen for several periods. The period is on the order of ten minutes, so it is recommended that you use a time-lapse photography method. Using this data, determine the period T.

Once you have sufficient data to determine the period, leave the apparatus and allow it to settle. Again, this settling requires more than a dozen periods, so you will likely want to leave the lab and come back on another day.

Measure the length L_0, and using your measurements and the equations above, estimate the value of G. Your estimate should have an error no larger than 20% from the literature value. Anything larger is unacceptable.

IV. APPENDIX

TABLE I: Table of measurements detailing the apparatus

Apparatus Specifications	
m_1	1.5 kg
d	50 mm
r	47 mm
K	1.083

[1] LEYBOLD Physics Leaflets *http://www.hep.fsu.edu/ wahl/phy3802/expinfo/cavendish /P1131_e.pdf*

[2] C. J. Foot, *Atomic Physics* (Oxford University Press, New York, 2005)